March 2nd, 2015 **Name (Please Print)** ____________________

Analysis II - Midterm I - Semester II 14/15

Instructions: *You may use any result stated in class but please state it precisely before using the same. You are not allowed to use any result proved in the H.W.*

Max Score: 50 **Time:** 2 hours and 30 minutes.

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1. (10 points) Let I be a bounded interval. Let $f, g : I \to \mathbb{R}$ be bounded continuous Reimann integrable functions. Suppose $f \leq g$ and $\int_I f = \int_I g$ then show that $f = g$ on I.

2. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous function. Let $F : [0, 1] \to \mathbb{R}$ be given by

$$F(x) = \int_{[x^2, x]} f.$$

Decide whether F is differentiable on $(0, 1)$ and if it is find F'.

3. Let $f : [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \ \ n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

(a) (5 points) Show that f is Riemann integrable in $[a, 1]$ for any $0 < a < 1$.
(b) (10 points) Is f Reimann integrable on $[0, 1]$?

4. (15 points) Let

$$U = \left\{ \begin{bmatrix} x \\ g(x) \end{bmatrix} : 0 \leq x \leq 1 \right\}$$

be a subset of $\mathbb{R}^2$ where $g : \mathbb{R} \to \mathbb{R}$ is given by

$$g(x) = \begin{cases} \frac{1}{x^2} \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Find the limit points of U.